

Differential Equations (MAS118) Problem Sheet 9

Problem 25

Determine the general solution of the following systems of linear differential equations

a) $y_1' = -y_1 + 6y_2, \quad y_2' = -3y_1 + 8y_2$ (6 points)

b) $y_1' = -y_1 + y_2, \quad y_2' = y_1 - y_2$ (6 points)

c) $y_1' = -4y_1 - 8y_2, \quad y_2' = 4y_1 + 4y_2$ (6 points)

d) $y_1' = -y_1 + 3y_2, \quad y_2' = -3y_1 - y_2$ (6 points)

e) $y_1' = -6y_1 + 4y_2, \quad y_2' = -8y_1 + 2y_2$ (6 points)

Problem 26

Determine the solution of the following initial value problems and sketch each solution in a diagram

a) $y_1' = -8y_1 - 10y_2, \quad y_2' = 5y_1 + 7y_2, \quad y_1(0) = 3, \quad y_2(0) = 1$ (16 points)

b) $y_1' = -6y_1 + y_2, \quad y_2' = -3y_2, \quad y_1(0) = 1, \quad y_2(0) = 0$ (16 points)

c) $y_1' = -y_1 + 5y_2, \quad y_2' = -5y_1 + y_2, \quad y_1(0) = 5, \quad y_2(0) = 5$ (16 points)

Problem 27

a) Compute the general solution of the linear second order differential equation

$$9y'' + 6y' + 19y = 0$$

(6 points)

b) Rewrite the second order differential equation as a linear system of first order differential equations and determine the general solution of this system of differential equations. (8 points)

c) Verify that the solutions of part a) and part b) coincide. (8 points)

Problem I

Rewrite the the system of first order differential equations

$$y_1' = 2y_1 - y_2, \quad y_2' = -y_1 - 3y_2$$

as a linear second order differential equation.

(20 bonus points)